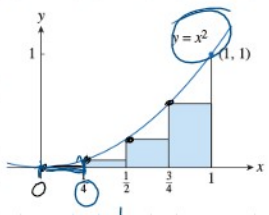
25-Jan 2023

Wednesday, January 25, 2023 2:18 PM

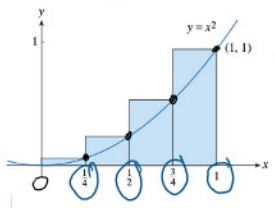


$\Delta x = \frac{1}{4}$

$bh + bh + bh + bh$

$L_4 \quad \frac{1}{4}f(0) + \frac{1}{4}f(\frac{1}{4}) + \frac{1}{4}f(\frac{1}{2}) + \frac{1}{4}f(\frac{3}{4})$

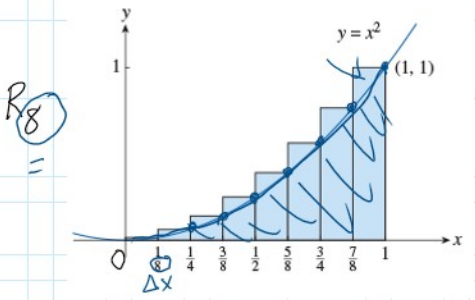
vs



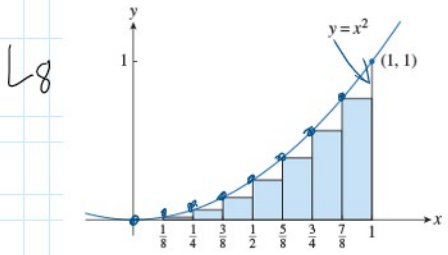
$R_4 = \frac{1}{4}\left(f(\frac{1}{4}) + f(\frac{1}{2}) + f(\frac{3}{4}) + f(1)\right)$

Δx = sum of heights

$(\frac{1}{4})^2$

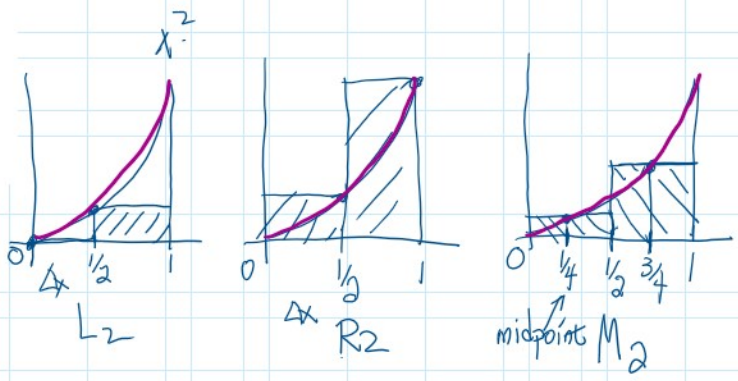


R_8 =

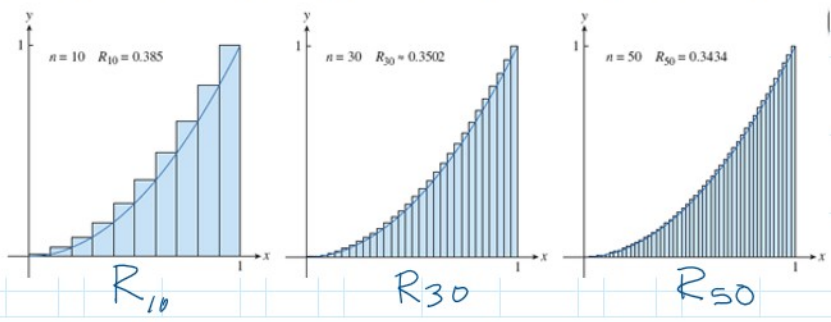


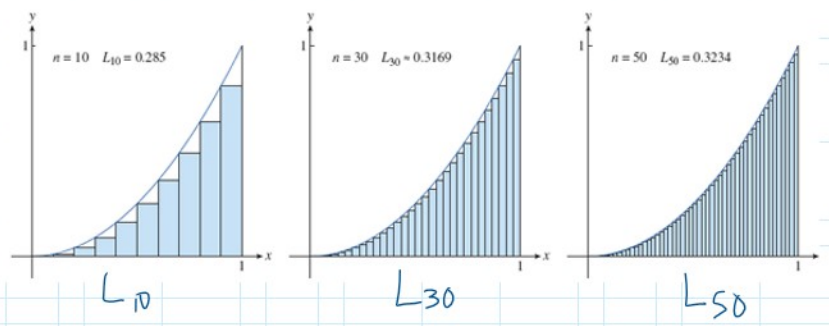
L_8

L_2 vs R_2 vs M_2



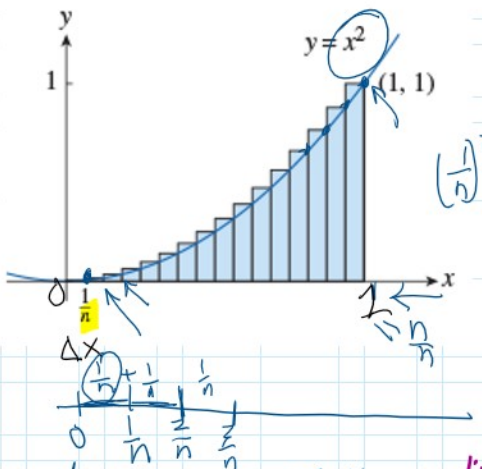
$f(x) = x^2$





long term solution:
use infinitely many rectangles
to find EXACT area under curve between $x=a$ and $x=b$

explore R_n as $n \to \infty$



$R_n = \Delta x$ (sum of heights)

$= \frac{1}{n}\left(\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \left(\frac{3}{n}\right)^2 + \dots + \left(\frac{n}{n}\right)^2\right)$

$\left(\frac{1}{n}\right)^2 = \frac{1^2}{n^2} = 1^2 \cdot \frac{1}{n^2}$

$= \frac{1}{n}\left(1^2 \cdot \frac{1}{n^2} + 2^2 \cdot \frac{1}{n^2} + 3^2 \cdot \frac{1}{n^2} + \dots + n^2 \cdot \frac{1}{n^2}\right)$

$= \frac{1}{n} \cdot \frac{1}{n^2}\left(1^2 + 2^2 + 3^2 + \dots + n^2\right)$

$= \frac{1}{n^3} \cdot \sum_{i=1}^{n} i^2$

$= \frac{1}{n^3/2} \cdot \frac{n(n+1)(2n+1)}{6}$ simplify

recall: Limit Laws:

$\lim_{x \to \infty}\left[f(x) \cdot g(x)\right] = \lim_{x \to \infty} f(x) \cdot \lim_{x \to \infty} g(x)$

$\lim_{n \to \infty} R_n$

$= \lim_{n \to \infty} \frac{(n+1)(2n+1)}{6n^2}$ (leading terms: $\frac{2n^2}{6n^2}$)

$= \frac{1}{6} \lim_{n \to \infty} \frac{n+1}{n} \cdot \frac{2n+1}{n}$

$= \frac{1}{6} \lim_{n \to \infty} \frac{n+1}{n} \cdot \lim_{n \to \infty} \frac{2n+1}{n}$

$= \frac{1}{6} \cdot 1 \cdot 2 = \frac{2}{6} = \frac{1}{3}$

exact area under $f(x) = x^2$ btwn $x=0$ and $x=1$

n boxes

$\lim_{n \to \infty} R_n = \lim_{n \to \infty} \frac{(n+1)(2n+1)}{6n^2}$

$\lim_{n \to \infty} \frac{2n^2 + 3n + 1}{6n^2}$

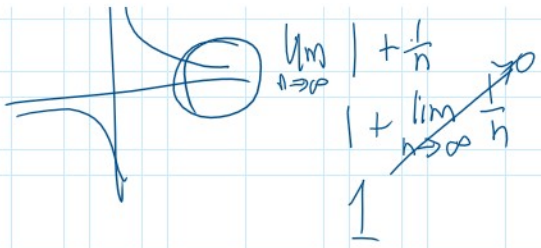
$\lim_{n \to \infty} \frac{2n^2}{6n^2} = \frac{1}{3}$

exact area of x^2 from $x=0$ to $x=1$ is $\frac{1}{3}$

$\lim_{x \to \infty} \frac{3x^2 + x - 7}{4x^2 - x + 1}$

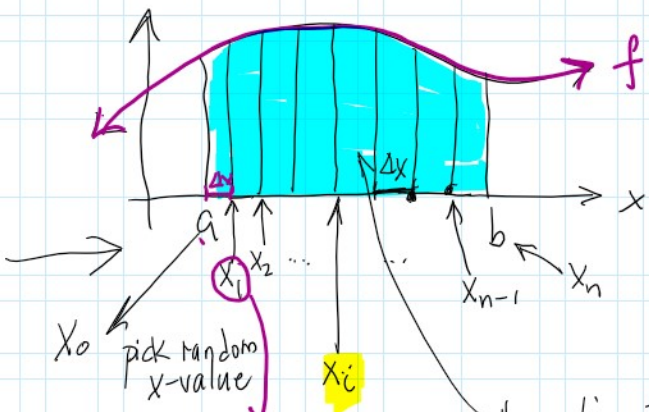
$\frac{n}{n} + \frac{1}{n}$

$\lim_{n \to \infty} 1 + \frac{1}{n}$ $\to 0$



btwn $x=0$ and $x=1$

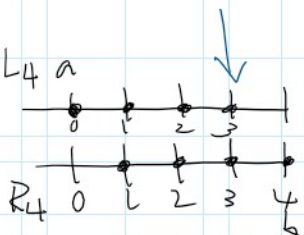
GENERALIZING FURTHER:



Δx is base of each slice

n slices

$$\Delta x = \frac{b-a}{n}$$



$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \Delta x\left(f(x_1)+f(x_2)+\dots+f(x_{n-1})+f(x_n)\right)$$

OR

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} \Delta x\left(f(x_0)+f(x_1)+\dots+f(x_{n-2})+f(x_{n-1})\right)$$

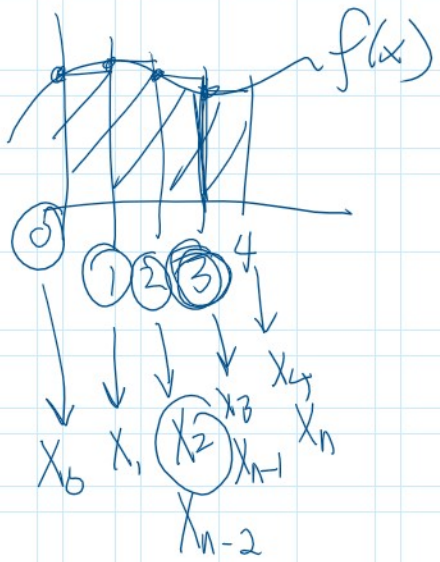
$x_1 = a+\Delta x$

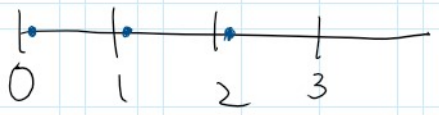
$x_2 = a+2\Delta x$

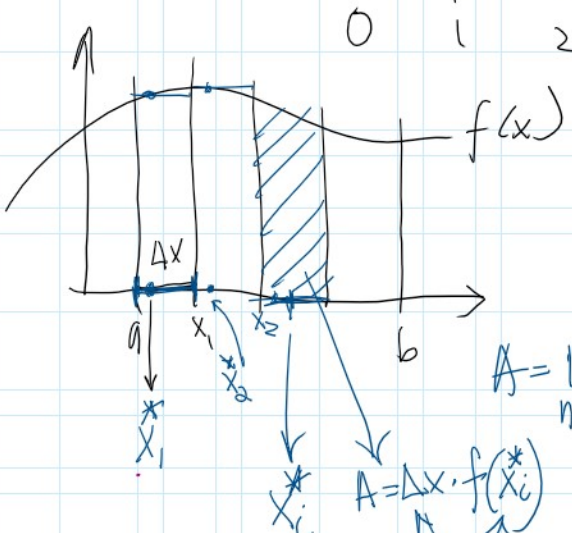
$x_3 = a+3\Delta x$

$\vdots$

$x_i = a+i\cdot\Delta x$



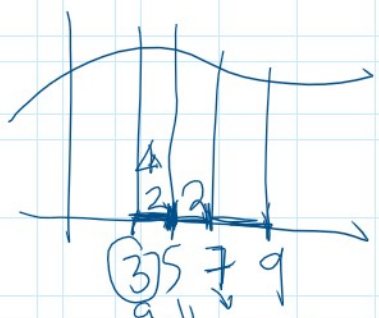




$$A = \lim_{n\to\infty} \Delta x\left(f(x_1^*)+f(x_2^*)+\dots+f(x_n^*)\right)$$

write in sigma notation

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\cdot\Delta x$$



$$\Delta x = \frac{9-3}{3} = \frac{6}{3} = 2$$

$a+2\Delta x$

$$\lim_{x\to\infty} \frac{x^3}{x^2} = \frac{\infty^3}{\infty^2} = \infty$$

$$\lim_{x\to\infty} \frac{x^2}{x^3} = \frac{\infty^2}{\infty^3} = \frac{1}{\infty}$$

$\frac{1}{\infty} \to 0$

$$\lim_{n\to\infty} \frac{n+1}{n} = 1$$

$$\lim_{n\to\infty} \frac{2n+1}{n} = 2$$

$$\lim_{x\to\infty} \frac{3x^2}{4x^2} = \frac{3}{4}$$

$f(x) = \frac{1}{x}$ ~~find~~ estimate area under curve @ $x=1 \Rightarrow x=7$

w 3